ON LARGE N FIXED POINTS OF A U(N) SYMMETRIC $(\varphi^*\cdot\varphi)^3_{D=3}$ MODEL COUPLED TO FERMIONS

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The three-dimensional U(N) symmetric $\eta(\varphi^*\cdot\varphi)^3$ model coupled to N-component fermions is considered within the $1/N$ expansion. In contrast to the purely bosonic case, here we find in the large N limit only a (nonperturbative) ultraviolet fixed point at $\eta = \eta^* \approx 179$, whereas infrared fixed points are absent.

Several years ago the O(N) (U(N)) symmetric $\eta(\varphi^*\cdot\varphi)^3$ model in $D=3$ (euclidean space) dimensions (we shall consider explicitly the complex case),

$$\mathcal{L}_\varphi = |\partial_\mu\varphi|^2 + M^2\varphi^*\cdot\varphi + (\lambda/2N)(\varphi^*\cdot\varphi)^2 + (\eta/3N^2)(\varphi^*\cdot\varphi)^3,$$

$$\varphi^*\cdot\varphi \equiv \sum_{a=1}^{N} \varphi_a^*\varphi_a, \qquad (1)$$

came under intensive investigation both in the context of critical phenomena (as describing logarithmic corrections to scaling near a tricritical point) [1], as well as in the issues of symmetry breaking and vacuum stability [2]. Recently there was a renewed interest in (1) [3] mainly due to the establishment of a nonperturbative ultraviolet fixed point (in the large N limit) bearing crucial implications for the ground state structure, the spectrum and the ultraviolet behavior of (1).

In the present note we propose to analyze a generalization of (1):

$$\mathcal{L} = \mathcal{L}_\varphi + i\bar{\psi}\partial\!\!\!/\psi - (\bar{\psi}\cdot\varphi)\rho - \bar{\rho}(\varphi^*\cdot\psi), \qquad (2)$$

within the $1/N$ expansion in regard of nontrivial fixed points. In (2) ψ is an N-component fermion field and the auxiliary fermion field ρ is a Lagrange multiplier for the nonlinear constraints: $\varphi^*\cdot\psi = \bar{\psi}\cdot\varphi = 0$. Let us stress that the $\psi-\varphi$ interaction in (2) is the only possible renormalizable *and* P-invariant one (in $D=3$). Other renormalizable interactions $(\bar{\psi}\cdot\varphi)(\varphi^*\cdot\psi)$, $(\varphi^*\cdot\varphi)\times(\bar{\psi}\cdot\psi)$ change sign under P-reflection.

The model (2) arises as a simple particular case of the general class of $D=3$ $\mathrm{U}(n)_{\mathrm{gauge}} \otimes \mathrm{U}(N)_{\text{“flavor”}}$ Higgs models with fermions (including the supersymmetric Higgs, $(\Phi^*\cdot\Phi)^2_{D=3}$ and the corresponding supersymmetric (generalized) nonlinear sigma-models) which were shown to possess several interesting properties within the $1/N$ expansion [4]: a phase transition due to spontaneous breakdown of the internal symmetry, dynamical breakdown of discrete symmetries (P- and T-reflections), a nonperturbative particle spectrum (dynamical generation of (topological) gauge invariant mass terms [5] for the gauge fields and forming of composite fermions in the “high-temperature” phase, “confinement” of the gauge fields and some of the φ's and ψ's in the “low-temperature” phase). Also exact expressions (in the large N limit) of the renormalization group functions and the corresponding fixed point structure of the general $D=3$ Higgs model with fermions are derived in ref. [6]. In particular, the scale invariant theories at any infrared or ultraviolet fixed point (except for the gaussian one) were shown to be P-invariant and nonlinear (i.e. containing nonlinear constraints on φ,ψ). However, it turns out that (2) does not lie on any generic ultraviolet or infrared stable surface in the coupling constant space of the general model, thus necessitating a special treatment.

The $1/N$ expansion of (2) is constructed along the standard line [1] starting from its auxiliary field form:

$$\mathcal{L}' = |\partial_\mu \varphi|^2 + [\alpha_0 + \mathrm{i}\alpha(x)]\,[\varphi^*\cdot\varphi - N\mu/T - N\sigma(x)]$$
$$+ 4N\mu\,\mu^{-1}\sigma^2 + \tfrac{1}{3}N\eta\sigma^3 + \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - (\bar{\psi}\cdot\varphi)\rho - \bar{\rho}(\varphi^*\cdot\psi),$$
$$(M/\mu)^2 + \lambda/\mu T + \eta/T^2 = 0,$$
$$u = 8(\lambda/\mu + 2\eta/T)^{-1}. \qquad (3)$$

Here μ is a common mass scale and α_0 is an arbitrary fixed nonnegative constant (in the $1/N$ expansion $\alpha_0 = m_\varphi^2$, the physical φ-mass squared). To analyze fixed points we need (3) only on the critical surface $T = T_c$ (i.e. $m_\varphi = 0, \langle\varphi\rangle = 0$ [‡1]). The "free" propagators ($\langle...\rangle^{(0)}$) of the $1/N$ Feynman graphs read (in momentum space):

$$\langle\varphi_a\varphi_b^*\rangle^{(0)} = \delta_{ab}(p^2)^{-1}, \quad \langle\psi_a\bar{\psi}_b\rangle^{(0)} = \delta_{ab}p\!\!\!/(p^2)^{-1},$$
$$\langle\rho\bar{\rho}\rangle^{(0)} = N^{-1}16p\!\!\!/(p^2)^{-1/2},$$
$$\langle\alpha\alpha\rangle^{(0)} = N^{-1}8(p^2)^{1/2}[1 + (u/\mu)(p^2)^{1/2}]^{-1},$$
$$\langle\sigma\sigma\rangle^{(0)} = N^{-1}(u/8\mu)[1 + (u/\mu)(p^2)^{1/2}]^{-1},$$
$$\langle\alpha\sigma\rangle^{(0)} = N^{-1}\mathrm{i}(u/\mu)(p^2)^{1/2}[1 + (u/\mu)(p^2)^{1/2}]^{-1}. \qquad (4)$$

Graphically (see figs. 1a–1c) they are represented as solid, solid directed, double solid directed, dashed, dotted and dashed–dotted lines, respectively. All vertices can be read off directly from the non-quadratic part of (3).

The general form of the renormalization group equations for (3) is as follows:

$$\Big(\mu\partial/\partial\mu + (1 - 2\zeta_\sigma)u\partial/\partial u + \beta_\eta\partial/\partial\eta$$
$$+ \sum_\Omega \zeta_\Omega \int \mathrm{d}^3x\, j_\Omega(x)\delta/\delta j_\Omega(x)\Big) W[\{j_\Omega\}]$$
$$= \int \mathrm{d}^3x\,[\omega_\alpha j_\alpha^3(x) + \omega_{\alpha\sigma} j_\alpha(x) j_\sigma(x) + \omega_\rho \bar{j}_\rho(x) j_\rho(x)],$$
$$\{\Omega\} \equiv \{\varphi, \psi, \alpha, \sigma, \rho\},$$
$$\beta_\eta = N^{-1}\beta_\eta^{(1)} + \mathrm{O}(N^{-2}), \quad \zeta_\Omega = N^{-1}\zeta_\Omega^{(1)} + \mathrm{O}(N^{-2}), \qquad (5)$$

‡1 In the case of (2) (or (3)), unlike the general Higgs model with fermions, there is no generation of $m_\psi \neq 0$ and, consequently, no dynamical breakdown of P-invariance.

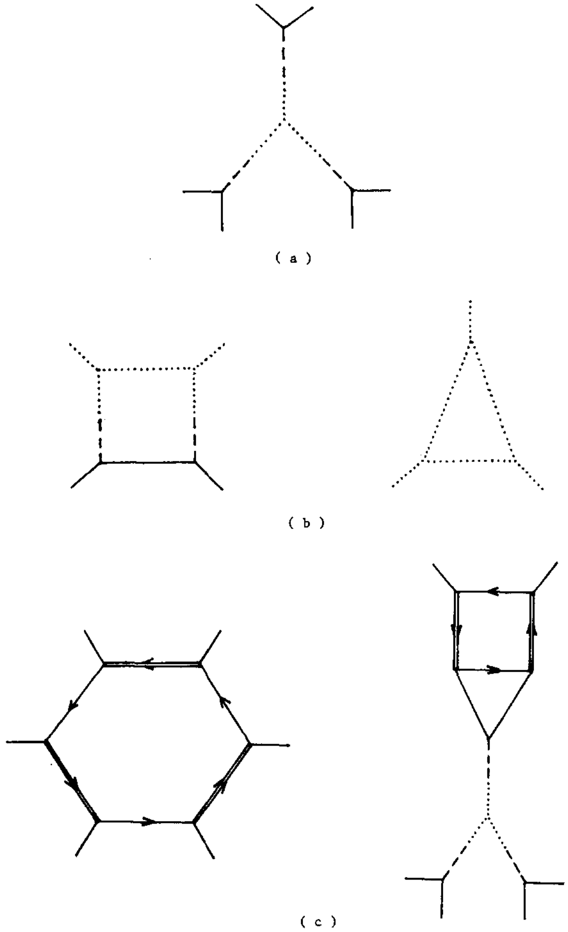

Fig. 1. $1/N$ graphs contributing to: (a) $\Gamma_{(0)}^{(6,0;...,0)}$; (b) $\Gamma_{(1)}^{(6,0;...,0)}$ without fermion loops; (c) $\Gamma_{(1)}^{(6,0;...,0)}$ containing fermion loops. Each trilinear vertex with attached dotted lines bears a factor $(-\eta)$.

where $W[\{j_\Omega\}]$ denotes the (renormalized) generating functional of connected Green's functions of (3). Here we employ a mass-independent renormalization scheme [7,4] of the "soft-mass" type [8] ("soft" masses are introduced only in $\langle\varphi\varphi^*\rangle^{(0)}, \langle\psi\bar{\psi}\rangle^{(0)}$ (4)). We shall use (5) in the leading $1/N$ order:

$$(\mu\partial/\partial\mu + u\partial/\partial u)\Gamma_{(1)}^{(L_\varphi,L_\psi;L_\alpha,L_\sigma,L_\rho)}$$
$$+ [\beta_\eta^{(1)}\partial/\partial\eta + \zeta_\sigma^{(1)}(L_\sigma - 2u\partial/\partial u) + \zeta_\alpha^{(1)}L_\alpha$$
$$+ \zeta_\rho^{(1)}L_\rho - \zeta_\varphi^{(1)}L_\varphi - \zeta_\psi^{(1)}L_\psi]\,\Gamma_{(0)}^{(L_\varphi,...,L_\rho)}$$
$$= \omega_\alpha^{(1)}\delta_{3L_\alpha} + \omega_{\alpha\sigma}^{(1)}\delta_{1L_\alpha}\delta_{1L_\sigma} + \omega_\rho^{(1)}\delta_{2L_\rho}, \qquad (6)$$

$$\Gamma^{(L_\varphi, L_\psi;\dots,L_\rho)} \equiv (\delta^{L_\varphi}/\delta\varphi\dots\delta\varphi^*)(\delta^{L_\psi}/\delta\psi\dots\delta\bar{\psi})\dots$$

$$\times\,(\delta^{L_\rho}/\delta j_\rho\dots\delta\bar{j}_\rho)\Gamma[\varphi,\psi\,|j_{\alpha,\sigma,\rho}]\,|_{\varphi=\psi=0,\,j_{\alpha,\sigma,\rho}=0},$$

$$\Gamma[\varphi,\psi\,|j_{\alpha,\sigma,\rho}] = W[\{j_\Omega\}] - \int d^3x\,[j^*_\varphi\varphi + \bar{j}_\psi\psi + \text{h.c.}],$$

$$\varphi(x) \equiv \delta W/\delta j^*_\varphi(x), \quad \psi(x) \equiv \delta W/\delta\bar{j}_\psi(x). \quad (6\text{ cont'd})$$

Here the subscripts (0), (1) of Γ denote leading, next-to-leading $1/N$ order, respectively. Now, from (6) by means of the $1/N$ graphical rules (4) and of the "soft-mass" renormalization scheme one can compute all renormalization group functions of (2) in the large N limit.

From eqs. (6) with $(L_\varphi, L_\psi;\dots,L_\rho) = (2,0;\dots,0)$, $(0,2;\dots,0),\dots,(0,\dots,2)$ we get:

$$\zeta^{(1)}_\varphi = (2/3\pi^2)[4 + I(u)], \quad \zeta^{(1)}_\psi = 8/3\pi^2,$$

$$\zeta^{(1)}_\alpha = -\zeta^{(1)}_\sigma = (16/3\pi^2)[2 - I(u)],$$

$$\zeta^{(1)}_\rho = -(2/3\pi^2)[8 + I(u)], \quad (7)$$

$$I(u) \equiv (1+u^2)^{-2}[1 + u^2 - \tfrac{1}{2}\pi(1-u^2) - 2u^2\ln u].$$

At this point we remark that as long as $\beta_u = u(1 - 2\zeta_\sigma)$ (cf. (5)), $u_* = 0$ (i.e. $\lambda_* = \infty$, cf. (3)) is an infrared fixed point exactly what is the case for (1). Moreover, η turns out to be irrelevant in the infrared scaling limit $u \to 0$ as it is for (1) (note from (7) that $\zeta^{(1)}_\Omega$ and $\beta^{(1)}_u$ $(= -2\zeta^{(1)}_\sigma)$ do not depend on η). Therefore, the interesting case, where one could eventually find new nontrivial fixed points for (2), is to set $u = \infty$ in (3), (4) (tricritical limit [1]).

Let us turn to eq. (6) with $(L_\varphi, L_\psi;\dots,L_\rho) = (6,0;\dots,0)$. All corresponding $1/N$ graphs (at $u = \infty$) are depicted in figs. 1a–1c. Simple calculations yield

$$\beta^{(1)}_\eta = 16^3/\pi^2 - (32/\pi^2)\eta + (3/4\pi^2)\eta^2$$

$$- (1/16^2\pi^2)\eta^3. \quad (8)$$

The sum of the third and the fourth terms on the right hand side of (8) exactly coincides with $\beta^{(1)}_\eta$ in the purely bosonic case (1) (see Townsend (1977) [2] and Appelquist and Heinz [3]). The crucial effect of the internal fermion loops in $\Gamma^{(6,\dots,0)}$ (fig. 1c) is the appearance of a large constant term in (8). Clearly, the origin $\eta = 0$ ceases to be (an infrared) fixed point (unlike (1)) and also, the ultraviolet fixed point occurs at a smaller value of $\eta = \eta^* \approx 179$, as compared to $\eta^* = 192$ for (1). Possible implications of this result will be discussed in a future paper.

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